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$b \to s\gamma$ Decay and Right-handed Top-bottom Charged Current

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Abstract

We introduce an anomalous top quark coupling (right-handed current) into Standard Model Lagrangian. Based on this, a more complete calculation of $b \to s\gamma$ decay including leading log QCD corrections from m_{top} to M_W in addition to corrections from M_W to m_b is given. The inclusive decay rate is found to be suppressed comparing with the case without QCD running from m_t to M_W except at the time of small values of $|f_R^{tb}|$. e.g. when $f_R^{tb} = -0.08$, it is only 1/10 of the value given before. As $|f_R^{tb}|$ goes smaller, this contribution is an enhancement like standard model case. From the newly experiment of CLEO Collaboration, strict restrictions to parameters of this top-bottom quark coupling are found.

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1 Introduction

The standard model(SM) has achieved great success recent years. However, there is still a vast interest beyond standard model. It is well known that the process $b \to s\gamma$ is extremely sensitive to new physics beyond the Standard Model[6]. It has been argued that this experiment provides more information about restrictions on the Standard Model, 2-Higgs doublet model, Supersymmetry, Technicolor and etc. Since the top quark is much heavier than other fermions, and its interactions may be quite sensitive to new physics, the interactions of the top quark are of special importance. In ref.[1], a right-handed coupling of the top-bottom charged current is added to the standard model(SM). Based on this, the authors give out the constraints to right-handed top quark current by $b \to s\gamma$ decay. In fact, the right handed current is also a low energy phenomena of $SU(2)_L \times SU(2)_R \times U(1)$ left-right symmetric model[2]. In this model, the right handed charged current is induced by the $W_L - W_R$ mixing in addition to the usual left-handed current in the standard model. Interference of the right- and left-handed currents in the penguin diagram enables a chirality-flip by the top quark mass m_t inside the loop, and therefore leads to an amplitude proportional to m_t[3, 4, 5]. (In standard model, the amplitude is proportional to the mass of the bottom or strange quark, because the pure (V-A) structure of the charged currents requires the chirality-flip to proceed only through the mass of the initial or the final state quark.) Therefore, large contributions from top-bottom quark right-handed current to $b \to s\gamma$ amplitude occur. To first order approximation, this just like to add a right-handed current to top-bottom quark coupling in SM.

Recently the CLEO Collaboration has measured the inclusive branching ratio of $b \to s\gamma$ to be[7]

$$Br(b \to s\gamma) = (2.32 \pm 0.51 \pm 0.29 \pm 0.32) \times 10^{-4}. \tag{1}$$

Corresponding to 95% confidence level, the range is $1 \times 10^{-4} < Br(b \to s\gamma) < 4 \times 10^{-4}$. This is a more stringent constraints compared with previous CLEO experiments[8] cited by the above papers[1, 4, 5]. Furthermore, with more precise experiments, a more accurate theoretical calculation of this decay rate is also needed.

The inclusive $b \to s\gamma$ decay rate is often assumed to be well described by the spectator model, where the b quark goes a radiative decay. The QCD corrections to this decay have been calculated to leading logarithmic accuracy by many authors[9, 10, 11, 12, 13, 14], and are known to enhance the decay rate within SM by a factor of 3-4. This enhancement, however, is subject to large uncertainties due to many reasons[15]. One of these reasons is the QCD running from top quark scale to W boson scale, which has been discussed in SM by ref.[16, 17], in 2-Higgs doublet model by ref.[18], and in Supersymmetry case by ref.[19]. This contribution which is usually considered as a next-leading order effect was found to give an additional enhancement up to 20% in the case of a much heavy top quark mass.

In our present paper, by introducing a right-handed charged current to top-bottom couplings, we recalculate the $b \to s\gamma$ decay including QCD running from m_{top} to M_W, in addition to corrections from M_W to m_b, in order to give a more complete leading log result in this model. Since the branching ratio is proportional to $|f_R^{tb} m_t/m_b|^2$, it is more complicated than that in SM or other models. Finally more recent CLEO experiment is used to give more precise restrictions to this anomalous top quark coupling.

In the next section, we first integrate out the top quark, generating an effective five-quark theory. By using the renormalization group equation, we run the effective field theory down to the W-scale where the weak bosons are removed. Then we continue running the effective field theory down to b-quark scale to include QCD corrections from M_W to m_b. In section 3, the rate of radiative b decay is obtained. Section 4 is a short summary.

2 QCD Corrections to $b \to s\gamma$ decay

In standard model and many other models, there is no flavor changing neutral current at tree level. It is only occurred through electroweak loop. In standard model Lagrangian, the relevant

charged current in R_ξ gauge reads

$$\begin{aligned}
\mathcal{L}_{CC} &= \frac{1}{\sqrt{2}}\mu^{\epsilon/2} g_2 \left(\begin{array}{ccc} \overline{u} & \overline{c} & \overline{t} \end{array} \right)_L \gamma_\mu V \left(\begin{array}{c} d \\ s \\ b \end{array} \right)_L W_+^\mu \\
&+ \frac{1}{\sqrt{2}} \frac{\mu^{\epsilon/2} g_2}{M_W} \left[\left(\begin{array}{ccc} \overline{u} & \overline{c} & \overline{t} \end{array} \right)_R M_U V \left(\begin{array}{c} d \\ s \\ b \end{array} \right)_L - \left(\begin{array}{ccc} \overline{u} & \overline{c} & \overline{t} \end{array} \right)_L V M_D \left(\begin{array}{c} d \\ s \\ b \end{array} \right)_R \right] \phi_+ \\
&+ h.c.. \qquad (2)
\end{aligned}$$

Where V represents the 3×3 unitary Kobayashi-Maskawa matrix, M_U and M_D denote the diagonalized quark mass matrices, the subscript L and R denote left-handed and right-handed quarks, respectively. This charged current, which is the main cause of flavor changing neutral current, is pure left-handed.

Like ref.[1], a phenomenological coupling of the right-handed top and bottom quarks to the W boson is introduced:

$$\mathcal{L} = g_2/\sqrt{2}\mu^{\epsilon/2} V_{tb} f_R^{tb} \overline{t}_R \gamma_\mu b_R W_+^\mu + \frac{\mu^{\epsilon/2} g_2}{\sqrt{2} M_W} V_{tb} f_R^{tb} \overline{t}(m_t P_R - m_b P_L) b \phi_+ + h.c., \qquad (3)$$

with f_R^{tb} denoting the strength of this additional coupling. Using this interaction (3) together with (2), we performed the QCD corrections to $b \to s\gamma$ decay.

At first, we integrate out the top quark, generating an effective five quark theory, introducing dimension-5 and dimension-6 effective operators as to include effects of the absent top quark. Higher dimension operators are suppressed by factor of p^2/m_t^2, here p^2 characterizes the interesting external momentum of b quark $p^2 \sim m_b^2$. For leading order of m_b^2/m_t^2, dimension-6 operators are good enough to make a complete basis of operators[16, 17][3]:

$$dimension\ 5: \qquad O_{LR}^1 = -\frac{1}{16\pi^2} m_b \overline{s}_L D^2 b_R,$$

[3] Notice here W_{LR}^2 is a new operator compared with SM case.

$$
\begin{aligned}
O^2_{LR} &= \mu^{\epsilon/2}\frac{g_3}{16\pi^2}m_b\overline{s}_L\sigma^{\mu\nu}X^a b_R G^a_{\mu\nu}, \\
O^3_{LR} &= \mu^{\epsilon/2}\frac{eQ_b}{16\pi^2}m_b\overline{s}_L\sigma^{\mu\nu}b_R F_{\mu\nu}, \\
Q_{LR} &= \mu^{\epsilon}g_3^2 m_b\phi_+\phi_-\overline{s}_L b_R, \\
W^1_{LR} &= -i\mu^{\epsilon}g_3^2 m_b W^{\nu}_+W^{\mu}_-\overline{s}_L\sigma_{\mu\nu}b_R, \\
W^2_{LR} &= \mu^{\epsilon}g_3^2 m_b W^{\mu}_+W^{\mu}_-\overline{s}_L b_R,
\end{aligned}
$$

$dimension$ 6 :

$$
\begin{aligned}
P^{1,A}_L &= -\frac{i}{16\pi^2}\overline{s}_L T^A_{\mu\nu\sigma}D^\mu D^\nu D^\sigma b_L, \\
P^2_L &= \mu^{\epsilon/2}\frac{eQ_b}{16\pi^2}\overline{s}_L\gamma^\mu b_L\partial^\nu F_{\mu\nu}, \\
P^4_L &= i\mu^{\epsilon/2}\frac{eQ_b}{16\pi^2}\tilde{F}_{\mu\nu}\overline{s}_L\gamma^\mu\gamma^5 D^\nu b_L, \\
R^1_L &= i\mu^{\epsilon}g_3^2\phi_+\phi_-\overline{s}_L \not{D} b_L, \\
R^2_L &= i\mu^{\epsilon}g_3^2(D^\sigma\phi_+)\phi_-\overline{s}_L\gamma_\sigma b_L, \\
W^1_L &= i\mu^{\epsilon}g_3^2 W^{\nu}_+W^{\mu}_-\overline{s}_L\gamma_\mu \not{D}\gamma_\nu b_L, \\
W^2_L &= i\mu^{\epsilon}g_3^2(D^\sigma W^{\nu}_+)W^{\mu}_-\overline{s}_L\gamma_\mu\gamma_\sigma\gamma_\nu b_L, \\
W^3_L &= i\mu^{\epsilon}g_3^2 W_{+\mu}W^{\mu}_-\overline{s}_L \overleftrightarrow{\not{D}}\, b_L, \\
W^4_L &= i\mu^{\epsilon}g_3^2 W^{\nu}_+W^{\mu}_-\overline{s}_L(\overleftrightarrow{D}_\mu\gamma_\nu + \gamma_\mu\overleftrightarrow{D}_\nu)b_L. \qquad (4)
\end{aligned}
$$

Where $\overline{s}_L\overleftrightarrow{D}_\mu\gamma_\nu b_L$ stands for $(\overline{s}_L D_\mu\gamma_\nu b_L + (D_\mu\overline{s}_L)\gamma_\nu b_L)$ and the covariant derivative is defined as

$$
D_\mu = \partial_\mu - i\mu^{\epsilon/2}g_3 X^a G^a_\mu - i\mu^{\epsilon/2}eQA_\mu,
$$

with g_3 denoting the QCD coupling constant. The tensor $T^A_{\mu\nu\sigma}$ appearing in $P^{1,A}_L$ assumes the following Lorentz structure, the index A ranging from 1 to 4:

$$
\begin{aligned}
&T^1_{\mu\nu\sigma} = g_{\mu\nu}\gamma_\sigma, \quad T^2_{\mu\nu\sigma} = g_{\mu\sigma}\gamma_\nu, \\
&T^3_{\mu\nu\sigma} = g_{\nu\sigma}\gamma_\mu, \quad T^4_{\mu\nu\sigma} = -i\epsilon_{\mu\nu\sigma\tau}\gamma^\tau\gamma_5.
\end{aligned} \qquad (5)
$$

In our following calculations, we try to work in a background field R_ξ gauge (with $\xi = 1$)[20], in order to maintain explicit gauge invariance in calculations of anomalous dimensions. Furthermore the usually trilinear interaction between photon, W boson, and would-be Goldstone

boson vanish in this gauge. So that, we did not include operators which involve both W boson and would-be Goldstone boson in one operator, like $W_+^\nu \phi_- \overline{s}_L \gamma_\nu b_L$. With the above operators, we can write down our effective Hamiltonian as

$$\mathcal{H}_{eff} = 2\sqrt{2} G_F V_{tb} V_{ts}^* \sum_i C_i(\mu) O_i(\mu). \tag{6}$$

The coefficients $C_i(m_t)$ of operators can be calculated from full theory by matching conditions, keeping only leading orders of p^2/m_t^2[16, 17]. Terms proportional to $f_R^{tb} m_t/m_b$ are from right handed current. Since m_t/m_b is a very large value, it is very convenient for us to keep only leading orders of m_t/m_b. This is also the reason why only top-bottom right-handed charge current is introduced, while other up-down, charm-strange right-handed currents are ignored.

$$\begin{aligned}
C_{R_L^1} &= C_{R_L^2} = 1/g_3^2, \\
C_{Q_{LR}} &= -\left(1 - f_R^{tb} \frac{m_t}{m_b}\right)/g_3^2, \\
C_{W_{LR}^1} &= C_{W_{LR}^2} = \frac{m_t}{m_b} f_R^{tb} \frac{\delta}{g_3^2}, \\
C_{W_L^1} &= C_{W_L^2} = \delta/g_3^2, \\
C_{W_L^3} &= C_{W_L^4} = 0.
\end{aligned} \tag{7}$$

The other coefficients of operators are all from the integrations of electroweak loops. Terms like $\log(\mu^2/m_t^2)$ in coefficients of operators O_{LR}^3, P_L^2 and P_L^4 vanish here, because of the matching scale $\mu = m_t$. They will be regenerated at lower scales by renormalization group running of electroweak later. The other logarithms are all from the finite part integration of loops, for there are two different mass scale particles in one loop.

$$\begin{aligned}
C_{O_{LR}^1} &= -\left(\frac{1+\delta}{2(1-\delta)^2} + \frac{\delta}{(1-\delta)^3}\log\delta\right) + f_R^{tb}\frac{m_t}{m_b}\left(\frac{1-3\delta-4\delta^2}{2(1-\delta)^2} + \frac{\delta-4\delta^2}{(1-\delta)^3}\log\delta\right), \\
C_{O_{LR}^2} &= -\frac{1}{2}\left(\frac{1}{(1-\delta)} + \frac{\delta}{(1-\delta)^2}\log\delta\right)\left(1 - f_R^{tb}\frac{m_t}{m_b}\right), \\
C_{O_{LR}^3} &= \left(\frac{1}{(1-\delta)} + \frac{\delta}{(1-\delta)^2}\log\delta\right) + f_R^{tb}\frac{m_t}{m_b}\left(\frac{-1+6\delta}{(1-\delta)} + \frac{-\delta+12\delta^2}{(1-\delta)^2}\log\delta + 6\delta\log\frac{\mu^2}{m_t^2}\right), \\
C_{P_L^{1,1}} &= C_{P_L^{1,3}} = \left(\frac{\frac{11}{18}+\frac{5}{6}\delta-\frac{2}{3}\delta^2+\frac{2}{9}\delta^3}{(1-\delta)^3} + \frac{\delta+\delta^2-\frac{5}{3}\delta^3+\frac{2}{3}\delta^4}{(1-\delta)^4}\log\delta\right),
\end{aligned}$$

$$
\begin{aligned}
C_{P_L^{1,2}} &= \left(\frac{-\frac{8}{9}-\frac{1}{6}\delta+\frac{17}{6}\delta^2-\frac{7}{9}\delta^3}{(1-\delta)^3}+\frac{-\delta+\frac{10}{3}\delta^3-\frac{4}{3}\delta^4}{(1-\delta)^4}\log\delta\right),\\
C_{P_L^{1,4}} &= \left(\frac{\frac{1}{2}-\delta-\frac{1}{2}\delta^2+\delta^3}{(1-\delta)^3}+\frac{\delta-3\delta^2+2\delta^3}{(1-\delta)^4}\log\delta\right),\\
C_{P_L^2} &= \frac{1}{Q_b}\left(\frac{\frac{3}{4}+\frac{1}{2}\delta-\frac{7}{4}\delta^2+\frac{1}{2}\delta^3}{(1-\delta)^3}-\frac{1}{3}\delta+\left(\frac{\frac{1}{6}+\frac{5}{6}\delta-\frac{5}{3}\delta^3+\frac{2}{3}\delta^4}{(1-\delta)^4}-\frac{1}{6}-\frac{1}{3}\delta\right)\log\delta\right)\\
&\quad -\frac{1}{2}\log\frac{\mu^2}{m_t^2}-\delta\log\frac{\mu^2}{m_t^2}, \qquad (8)\\
C_{P_L^4} &= \frac{1}{Q_b}\left(\frac{-\frac{1}{2}-5\delta+\frac{17}{2}\delta^2-3\delta^3}{(1-\delta)^3}+\frac{-5\delta+7\delta^2-2\delta^3}{(1-\delta)^4}\log\delta+4\delta\log\delta\right)+12\delta\log\frac{\mu^2}{m_t^2},
\end{aligned}
$$

where $\delta = M_W^2/m_t^2$. When $f_R^{tb}=0$, the above results (7), (8) reduce to that of SM case[17].

The renormalization group equation satisfied by the coefficient functions $C_i(\mu)$ is

$$\mu\frac{d}{d\mu}C_i(\mu)=\sum_j(\gamma^\tau)_{ij}C_j(\mu). \qquad (9)$$

The solution to this renormalization group equation (9) appears in obvious matrix notation as

$$C(\mu_2)=\left[\exp\int_{g_3(\mu_1)}^{g_3(\mu_2)}dg\frac{\gamma^T(g)}{\beta(g)}\right]C(\mu_1), \qquad (10)$$

where the anomalous dimension matrix γ_{ij} is calculated in practice by requiring renormalization group equations for Green functions with insertions of composite operators to be satisfied order by order in perturbation theory[16, 17].

$$
\gamma = \begin{array}{c c}
 & \begin{array}{ccccccc} O_{LR}^1 & O_{LR}^2 & O_{LR}^3 & P_L^{1,A} & P_L^2 & P_L^3 & P_L^4 \end{array} \\
\begin{array}{c} Q_{LR} \\ R_L^1 \\ R_L^2 \\ W_{LR}^1 \\ W_{LR}^2 \\ W_L^1 \\ W_L^2 \\ W_L^3 \\ W_L^4 \end{array} &
\left(\begin{array}{ccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & -1/2 & 0 & 0\\
0 & 0 & 6 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 12\\
0 & 0 & 0 & 0 & -1 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right)
\end{array}
16\pi^2\,\frac{g_3^2}{8\pi^2}. \qquad (11)
$$

These mixings are all between operators (Q,R,W) induced by tree-diagram and operators (O,P) induced by loop-diagram. In order to see how the renormalization group method is accomplished, we neglect the proper QCD corrections for the moment, so that we can take into account only the above entries of anomalous dimensions. Insert this matrix to eqn.(10), we find the following relations:

$$\begin{aligned}
C_{O^3_{LR}}(M_W) &= C_{O^3_{LR}}(m_t) + 6\frac{m_t}{m_b} f_R^{tb} \delta \log\frac{\mu^2}{m_t^2},\\
C_{P^2_L}(M_W) &= C_{P^2_L}(m_t) - \frac{1}{2}\log\frac{\mu^2}{m_t^2} - \delta\log\frac{\mu^2}{m_t^2},\\
C_{P^4_L}(M_W) &= C_{P^4_L}(m_t) + 12\delta\log\frac{\mu^2}{m_t^2}.
\end{aligned}$$

Here the renormalization group equation reproduces the $\log(\mu^2/m_t^2)$ terms in the coefficients of operators at equation (8) which vanished at $\mu = m_t$. This proves the consistence of the whole calculation.

The QCD anomalous dimensions for each of the operators in our basis are[16, 17]:

$$\gamma = \begin{array}{c} \\ O^1_{LR} \\ O^2_{LR} \\ O^3_{LR} \\ P^{1,1}_L \\ P^{1,2}_L \\ P^{1,3}_L \\ P^{1,4}_L \\ P^2_L \\ P^4_L \end{array}
\begin{array}{c}
\begin{array}{ccccccccc} O^1_{LR} & O^2_{LR} & O^3_{LR} & P^{1,1}_L & P^{1,2}_L & P^{1,3}_L & P^{1,4}_L & P^2_L & P^4_L \end{array}\\
\begin{pmatrix}
\frac{20}{3} & 1 & -2 & 0 & 0 & 0 & 0 & 0 & 0\\
-8 & \frac{2}{3} & \frac{4}{3} & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & \frac{16}{3} & 0 & 0 & 0 & 0 & 0 & 0\\
6 & 2 & -1 & \frac{2}{3} & 2 & -2 & -2 & 0 & 0\\
4 & \frac{3}{2} & 0 & -\frac{113}{36} & \frac{137}{18} & -\frac{113}{36} & -\frac{4}{3} & \frac{9}{4} & 0\\
2 & 1 & 1 & -2 & 2 & \frac{2}{3} & -2 & 0 & 0\\
0 & \frac{1}{2} & 2 & -\frac{113}{36} & \frac{89}{18} & -\frac{113}{36} & \frac{4}{3} & \frac{9}{4} & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & -\frac{4}{3} & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\end{array}
\frac{g_3^2}{8\pi^2}, \tag{12}$$

$$\gamma = \begin{array}{c c} & \begin{array}{ccccccccc} Q_{LR} & R_L^1 & R_L^2 & W_{LR}^1 & W_{LR}^2 & W_L^1 & W_L^2 & W_L^3 & W_L^4 \end{array} \\ \begin{array}{c} Q_{LR} \\ R_L^1 \\ R_L^2 \\ W_{LR}^1 \\ W_{LR}^2 \\ W_L^1 \\ W_L^2 \\ W_L^3 \\ W_L^4 \end{array} & \left(\begin{array}{ccccccccc} \frac{23}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{23}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{23}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 13 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{23}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{8}{3} & 0 & \frac{23}{3} & 0 & -\frac{8}{9} & \frac{16}{9} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{23}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{23}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{16}{9} & \frac{101}{9} \end{array} \right) \end{array} \frac{g_3^2}{8\pi^2}. \qquad (13)$$

Notice here only the anomalous dimensions relevant to W_{LR}^2 are new to the SM case. To proceed with such complicated anomalous dimensions with so many operators, the algebra computation code REDUCE is used. Combining the above three matrices (11, 12,13) into one large 18×18 matrix, we diagonalize it analytically to give 18 eigenvalues and 18 eigenvectors. Using equation (10), one can have the coefficients of operators at $\mu = M_W$. All analytic expressions of coefficients at this scale, which are functions of coefficients at $\mu = m_t$, are included in the appendix.

In order to continue running the basis operator coefficients down to lower scales, one must integrate out the weak gauge bosons and would-be Goldstone bosons at $\mu = M_W$ scale. From matching conditions, one finds the following relations between coefficient functions just below(-) and above(+) $\mu = M_W$[16, 17]:

$$\begin{aligned} C_{O_{LR}^1}(M_W^-) &= C_{O_{LR}^1}(M_W^+), \\ C_{O_{LR}^2}(M_W^-) &= C_{O_{LR}^2}(M_W^+), \\ C_{O_{LR}^3}(M_W^-) &= C_{O_{LR}^3}(M_W^+), \\ C_{P_L^{1,1}}(M_W^-) &= C_{P_L^{1,1}}(M_W^+) + 2/9, \\ C_{P_L^{1,2}}(M_W^-) &= C_{P_L^{1,2}}(M_W^+) - 7/9, \\ C_{P_L^{1,3}}(M_W^-) &= C_{P_L^{1,3}}(M_W^+) + 2/9, \end{aligned}$$

$$
\begin{aligned}
C_{P_L^{1,4}}(M_W^-) &= C_{P_L^{1,4}}(M_W^+) + 1, \\
C_{P_L^2}(M_W^-) &= C_{P_L^2}(M_W^+) - C_{W_L^2}(M_W^+) - 3/2, \\
C_{P_L^3}(M_W^-) &= C_{P_L^3}(M_W^+), \\
C_{P_L^4}(M_W^-) &= C_{P_L^4}(M_W^+) + 9.
\end{aligned} \tag{14}
$$

The operators involving Goldstone ϕ and W bosons are absent after this matching.

In addition to these, there are new four-quark operators from the matching[9, 12, 14]:

$$
\begin{array}{ll}
O_1 = (\overline{c}_{L\beta}\gamma^\mu b_{L\alpha})(\overline{s}_{L\alpha}\gamma_\mu c_{L\beta}), & O_2 = (\overline{c}_{L\alpha}\gamma^\mu b_{L\alpha})(\overline{s}_{L\beta}\gamma_\mu c_{L\beta}), \\
O_3 = (\overline{s}_{L\alpha}\gamma^\mu b_{L\alpha})\sum_q(\overline{q}_{L\beta}\gamma_\mu q_{L\beta}), & O_4 = (\overline{s}_{L\alpha}\gamma^\mu b_{L\beta})\sum_q(\overline{q}_{L\beta}\gamma_\mu q_{L\alpha}), \\
O_5 = (\overline{s}_{L\alpha}\gamma^\mu b_{L\alpha})\sum_q(\overline{q}_{R\beta}\gamma_\mu q_{R\beta}), & O_6 = (\overline{s}_{L\alpha}\gamma^\mu b_{L\beta})\sum_q(\overline{q}_{R\beta}\gamma_\mu q_{R\alpha}),
\end{array} \tag{15}
$$

with coefficients

$$
C_i(M_W) = 0, \quad i = 1, 3, 4, 5, 6, \quad C_2(M_W) = 1.
$$

Although only coefficient of O_2 is not zero, the other operators should also be included, for their mixing through renormalization group running.

To simplify the calculation and compare with the previous results, equations of motion(EOM)[21] is used to reduce all the remaining two-quark operators to the gluon and photon magnetic moment operators O_{LR}^2 and O_{LR}^3. Neglecting the strange quark mass in comparison with the bottom quark mass, we obtain the on-shell equivalence relations:

$$
\begin{aligned}
O_{LR}^1 &= P_L^{1,1} = -\frac{1}{2}O_{LR}^2 - \frac{1}{2}O_{LR}^3, \\
P_L^{1,2} &= -P_L^{1,4} = -\frac{1}{4}O_{LR}^2 - \frac{1}{4}O_{LR}^3, \\
P_L^{1,3} &= P_L^2 = 0, \\
P_L^4 &= -\frac{1}{4}O_{LR}^3.
\end{aligned} \tag{16}
$$

To be comparable with previous results without QCD corrections from m_{top} to M_W, operators O_{LR}^3, O_{LR}^2 are rewritten as O_7, O_8 like ref.[9, 12, 14],

$$
\begin{aligned}
O_7 &= (e/16\pi^2) m_b \overline{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}, \\
O_8 &= (g/16\pi^2) m_b \overline{s}_L \sigma^{\mu\nu} T^a b_R G_{\mu\nu}^a.
\end{aligned} \tag{17}
$$

Then the operator basis now consists of 8 operators. Using eqn.(14) (16) together with (24) in the appendix, the effective Hamiltonian appearing just below the W-scale is easily drawn out:

$$\begin{aligned}
\mathcal{H}_{eff} &= \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i C_i(M_W^-) O_i(M_W^-) \\
&\stackrel{EOM}{\longrightarrow} \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left\{ \sum_{i=1}^{6} C_i(M_W^-) O_i + C_7(M_W^-) O_7 + C_8(M_W^-) O_8 \right\}. \qquad (18)
\end{aligned}$$

with

$$\begin{aligned}
C_{O_8}(M_W^-) &= \left(\frac{\alpha_s(m_t)}{\alpha_s(M_W)}\right)^{\frac{14}{23}} \left\{ \frac{1}{2} C_{O^1_{LR}}(m_t) - C_{O^2_{LR}}(m_t) + \frac{1}{2} C_{P^{1,1}_L}(m_t) \right. \\
&\left. + \frac{1}{4} C_{P^{1,2}_L}(m_t) - \frac{1}{4} C_{P^{1,4}_L}(m_t) \right\} - \frac{1}{3}, \qquad (19)
\end{aligned}$$

$$\begin{aligned}
C_{O_7}(M_W^-) = \frac{1}{3} \left(\frac{\alpha_s(m_t)}{\alpha_s(M_W)}\right)^{\frac{16}{23}} & \left\{ C_{O^3_{LR}}(m_t) + 8 C_{O^2_{LR}}(m_t) \left[1 - \left(\frac{\alpha_s(M_W)}{\alpha_s(m_t)}\right)^{\frac{2}{23}} \right] \right. \\
+ \left[-\frac{9}{2} C_{O^1_{LR}}(m_t) - \frac{9}{2} C_{P^{1,1}_L}(m_t) - \frac{9}{4} C_{P^{1,2}_L}(m_t) + \frac{9}{4} C_{P^{1,4}_L}(m_t) \right] & \left[1 - \frac{8}{9} \left(\frac{\alpha_s(M_W)}{\alpha_s(m_t)}\right)^{\frac{2}{23}} \right] \\
& \left. - \frac{1}{4} C_{P^4_L}(m_t) + \frac{9}{23} 16\pi^2 C_{W^1_L}(m_t) \left[1 - \frac{\alpha_s(m_t)}{\alpha_s(M_W)} \right] \right\} - \frac{23}{36}. \qquad (20)
\end{aligned}$$

Since they are expressed by coefficients of operators at $\mu = m_t$ and QCD coupling α_s, it is convenient to utilize these formulas.

If QCD correction from m_t to M_W is neglected[by setting $\alpha_s(M_W) = \alpha_s(m_t)$ in the above equations (19),(20)], the results are reduced exactly to those the previous authors gave[1, 4].

The QCD corrections from $\mu = M_W$ to $\mu = m_b$ have been a subject of many papers[9, 10, 11, 12, 13, 14]. It has attracted great interest recent years, since most QCD corrections come from this stage. For a long time, there are some discrepancies in this calculation, for it is rather lengthy calculation involving 2-loop diagrams. Until recently, it is completely resolved, and a complete leading log result is given[14]. Here, we are free to utilize these results, so the coefficients of operators at $\mu = m_b$ scale are

$$C_7^{eff}(m_b) = \eta^{16/23} C_7(M_W) + \frac{8}{3}(\eta^{14/23} - \eta^{16/23}) C_8(M_W) + C_2(M_W) \sum_{i=1}^{8} h_i \eta^{a_i}. \qquad (21)$$

With $\eta = \alpha_s(M_W)/\alpha_s(m_b)$,

$$h_i = \Big(\frac{626126}{272277}, \ -\frac{56281}{51730}, \ -\frac{3}{7}, \ -\frac{1}{14}, \ -0.6494, \ -0.0380, \ -0.0186, \ -0.0057\Big),$$

$$a_i = \Big(\frac{14}{23}, \ \frac{16}{23}, \ \frac{6}{23}, \ -\frac{12}{23}\Big).$$

Here $m_b = 4.9\text{GeV}$ is used.

3 The $\overline{B} \to X_s\gamma$ decay rate

Following previous authors[9, 12, 14], one obtains the $\overline{B} \to X_s\gamma$ decay rate normalized to the quite well established semileptonic decay rate $BR(\overline{B} \to X_c e\overline{\nu})$.

$$BR(\overline{B} \to X_s\gamma)/BR(\overline{B} \to X_c e\overline{\nu}) \simeq \Gamma(b \to s\gamma)/\Gamma(b \to ce\overline{\nu}). \tag{22}$$

Then

$$\frac{BR(\overline{B} \to X_s\gamma)}{BR(\overline{B} \to X_c e\overline{\nu})} \simeq \frac{6\alpha_{QED}}{\pi g(m_c/m_b)}|C_7^{eff}(m_b)|^2 \left(1 - \frac{2\alpha_s(m_b)}{3\pi} f(m_c/m_b)\right)^{-1}, \tag{23}$$

where $g(m_c/m_b) \simeq 0.45$ and $f(m_c/m_b) \simeq 2.4$ correspond to the phase space factor and the one-loop QCD correction to the semileptonic decay, respectively[23]. The electromagnetic fine structure constant evaluated at the b quark scale takes value as $\alpha_{QED}(m_b) = 1/132.7$. If we take experimental result $Br(\overline{B} \to X_c e\overline{\nu}) = 10.8\%$[22], the branching ratios of $\overline{B} \to X_s\gamma$ is found. Compared with the previous results without QCD running from m_t to M_W[1], the branching ratio is more sensitive to the value of f_R^{tb}. It is no longer a pure enhancement like SM case. It is rather suppressed when $|f_R^{tb}|$ takes large value, for example, When $|f_R^{tb}| = -0.08$, the branching ratio is only about 1/10 of that with QCD correction from m_t to M_W neglected. While when $|f_R^{tb}|$ takes small value, the branching ratio is rather enhanced, especially when $|f_R^{tb}| = 0$, it is enhanced 11% corresponding to SM case. Since the right-handed current contribution is proportional to $|f_R^{tb}|^2$, the whole QCD correction to $b \to s\gamma$ decay is always a large enhancement.

In Fig 1, the branching ratio of $b \to s\gamma$ is displayed as functions of f_R^{tb} with $m_t = 174\text{GeV}$[24]. The three lines correspond to $\alpha_s(m_Z) = 0.107, \ 0.117, \ 0.127$. They are all parabolic lines, because

$Br(b \to s\gamma)$ is proportional to $|C_7(m_b)|^2$, and $C_7(m_b)$ is proportional to f_R^{tb}. That the three lines are very close to each other implies that the branching ratios are not sensitive to the values of α_s.

Fig.2 is branching ratios displayed as functions of f_R^{tb}, with different top quark masses 158GeV, 174GeV and 190GeV. The QCD coupling constant is $\alpha_s(m_Z) = 0.117$. Since contributions from right-handed current are proportional to $|f_R^{tb} m_t/m_b|^2$, the three lines in Fig.2 diverge in the region far from $f_R^{tb} = 0$, but nearly converge to a same line when $f_R^{tb} \to 0$.

With the recent CLEO experiments of $b \to s\gamma$ decay, a new constraint to right-handed current can be found. From Fig.1 and Fig.2, the parameter f_R^{tb} is constrained in two small windows $-0.087 < f_R^{tb} < -0.050$ and $-0.023 < f_R^{tb} < 0.002$. Compared with results without QCD corrections from m_t to M_W[1], the whole line transfers to left side(towards minus). So the windows also transfer to minus side. Most allowed values of f_R^{tb} are negative.

4 Conclusion

In the above, we have introduced an anomalous right-handed current to top, bottom quarks and W boson coupling in the SM, and given the full leading log QCD corrections(including QCD running from m_{top} to M_W) to $b \to s\gamma$ decay in this model.

The QCD corrections from m_{top} to M_W to $b \to s\gamma$ decay enhance the decay rate in small values of f_R^{tb} like in SM case; but suppress the decay rate when f_R^{tb} takes larger value. The whole QCD correction still makes a large enhancement. The restrictions from $b \to s\gamma$ decay to anomalous top quark coupling are strict, only two narrow windows are allowed. It is shown that the decay $b \to s\gamma$ has been the most restrictive process so far in constraining the parameters of the right-handed current of top quark.

If a complete QCD next-leading log result of $b \to s\gamma$ decay is performed, one can expect to obtain more precise results from $b \to s\gamma$ decay by freely using our above results.

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A Operator coefficients at $\mu = M_W^+$ scale

We quote here the results of the coefficients of operators at $\mu = M_W^+$ scale. The W bosons and would-be Goldstone bosons are not integrated out yet.

$$
\begin{aligned}
C_{O^1_{LR}}(M_W) &= \left(2\zeta^{14/23} - \zeta^{8/23}\right) C_{O^1_{LR}}(m_t) + 4\left(-\zeta^{14/23} + \zeta^{8/23}\right) C_{O^2_{LR}}(m_t) \\
&\quad + \left(2\zeta^{14/23} - \zeta^{8/23} - \zeta^{-4/23}\right) C_{P^{1,1}_L}(m_t) \\
&\quad + \left(\zeta^{14/23} - \frac{89}{130}\zeta^{8/23} - \frac{113}{274}\zeta^{-4/23} + \frac{864}{8905}\zeta^{113/138}\right) C_{P^{1,2}_L}(m_t) \\
&\quad + \left(-\zeta^{14/23} + \frac{171}{130}\zeta^{8/23} - \frac{113}{274}\zeta^{-4/23} + \frac{864}{8905}\zeta^{113/138}\right) C_{P^{1,4}_L}(m_t), \\
C_{O^2_{LR}}(M_W) &= \frac{1}{2}\left(\zeta^{14/23} - \zeta^{8/23}\right) C_{O^1_{LR}}(m_t) + \left(-\zeta^{14/23} + 2\zeta^{8/23}\right) C_{O^2_{LR}}(m_t) \\
&\quad + \frac{1}{2}\left(\zeta^{14/23} - \zeta^{-4/23}\right) C_{P^{1,1}_L}(m_t) \\
&\quad + \left(\frac{1}{4}\zeta^{14/23} - \frac{6}{65}\zeta^{8/23} - \frac{113}{548}\zeta^{-4/23} + \frac{432}{8905}\zeta^{113/138}\right) C_{P^{1,2}_L}(m_t) \\
&\quad + \left(-\frac{1}{4}\zeta^{14/23} + \frac{53}{130}\zeta^{8/23} - \frac{113}{548}\zeta^{-4/23} + \frac{432}{8905}\zeta^{113/138}\right) C_{P^{1,4}_L}(m_t), \\
C_{O^3_{LR}}(M_W) &= \left(5\zeta^{14/23} - \frac{1}{2}\zeta^{8/23} - \frac{9}{2}\zeta^{16/23}\right) C_{O^1_{LR}}(m_t) + \left(-10\zeta^{14/23} + 2\zeta^{8/23} + 8\zeta^{16/23}\right) C_{O^2_{LR}}(m_t) \\
&\quad + \zeta^{16/23} C_{O^3_{LR}}(m_t) + \left(5\zeta^{14/23} - \frac{1}{2}\zeta^{-4/23} - \frac{9}{2}\zeta^{16/23}\right) C_{P^{1,1}_L}(m_t) \\
&\quad + \left(\frac{5}{2}\zeta^{14/23} - \frac{6}{65}\zeta^{8/23} - \frac{113}{548}\zeta^{-4/23} + \frac{432}{8905}\zeta^{113/138} - \frac{9}{4}\zeta^{16/23}\right) C_{P^{1,2}_L}(m_t) \\
&\quad + \left(-\frac{5}{2}\zeta^{14/23} + \frac{53}{130}\zeta^{8/23} - \frac{113}{548}\zeta^{-4/23} + \frac{432}{8905}\zeta^{113/138} + \frac{9}{4}\zeta^{16/23}\right) C_{P^{1,4}_L}(m_t) \\
&\quad + \frac{1}{4}\left(1 - \zeta^{16/23}\right) C_{P^4_L}(m_t) + \frac{18}{23}\left(1 - \zeta^{-1}\right) 16\pi^2 C_{W^1_{LR}}(M_W), \\
C_{P^{1,1}_L}(M_W) &= C_{P^{1,3}_L} = \zeta^{-4/23} C_{P^{1,1}_L}(m_t) + \frac{113}{274}\left(\zeta^{-4/23} - \zeta^{113/138}\right)\left(C_{P^{1,1}_L}(m_t) + C_{P^{1,4}_L}(m_t)\right), \\
C_{P^{1,2}_L}(M_W) &= \left(\zeta^{8/23} - \zeta^{-4/23}\right) C_{P^{1,1}_L}(m_t) + \left(\frac{1}{2}\zeta^{8/23} - \frac{113}{274}\zeta^{-4/23} + \frac{125}{137}\zeta^{113/138}\right) C_{P^{1,2}_L}(m_t) \\
&\quad + \left(-\frac{1}{2}\zeta^{8/23} - \frac{113}{274}\zeta^{-4/23} + \frac{125}{137}\zeta^{113/138}\right) C_{P^{1,4}_L}(m_t),
\end{aligned}
$$

$$
\begin{aligned}
C_{P_L^{1,4}}(M_W) &= \left(-\zeta^{8/23}+\zeta^{-4/23}\right) C_{P_L^{1,1}}(m_t) + \left(-\frac{1}{2}\zeta^{8/23}+\frac{113}{274}\zeta^{-4/23}+\frac{12}{137}\zeta^{113/138}\right) C_{P_L^{1,2}}(m_t) \\
&\quad + \left(\frac{1}{2}\zeta^{8/23}+\frac{113}{274}\zeta^{-4/23}+\frac{12}{137}\zeta^{113/138}\right) C_{P_L^{1,4}}(m_t), \\
C_{P_L^2}(M_W) &= \frac{81}{226}\left(\zeta^{113/138}-1\right)\left(C_{P_L^{1,2}}(m_t)+C_{P_L^{1,4}}(m_t)\right) + C_{P_L^2}(m_t) \\
&\quad + \frac{3}{46}(1-\zeta)(2\delta+1)16\pi^2/g_3^2(m_t), \\
C_{P_L^4}(M_W) &= C_{P_L^4}(m_t) + \frac{36}{23}(\zeta-1)16\pi^2\delta/g_3^2(m_t), \\
C_{W_{LR}^1}(M_W) &= \zeta^{39/23} C_{W_{LR}^1}(m_t) + \frac{1}{2}\left(\zeta-\zeta^{39/23}\right)\delta/g_3^2(m_t), \\
C_{W_L^3}(M_W) &= \zeta\left(C_{W_L^3}(m_t)+\frac{1}{4}(\zeta^{32/69}-1)(C_{W_L^1}(m_t)+2C_{W_L^4}(m_t))\right), \\
C_{W_L^4}(M_W) &= \zeta^{101/69}\left(C_{W_L^4}(m_t)+\frac{1}{2}(1-\zeta^{-32/69})C_{W_L^1}(m_t)\right), \\
C_{O_i}(M_W) &= \zeta C_{O_i}(m_t), \qquad O_i = Q_{LR},\ R_L^1,\ R_L^2,\ W_{LR}^2,\ W_L^1,\ W_L^2.
\end{aligned} \tag{24}
$$

Here $\zeta = \alpha_s(m_t)/\alpha_s(M_W)$. The factor of $16\pi^2$ in some of these expressions arises from mixing between operators induced by tree diagrams and those by loop diagrams.

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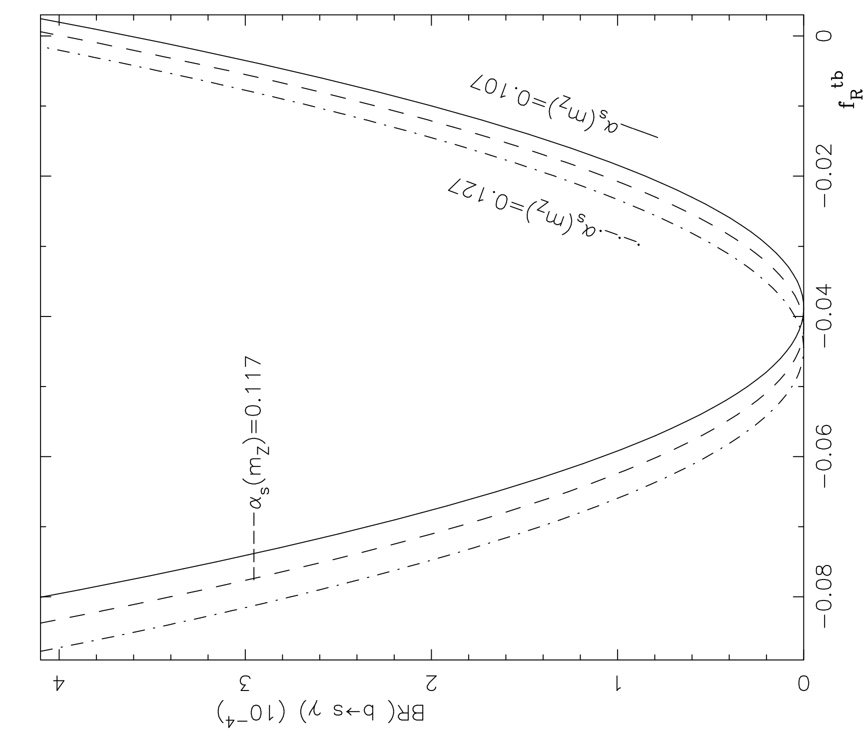
Figure Captions

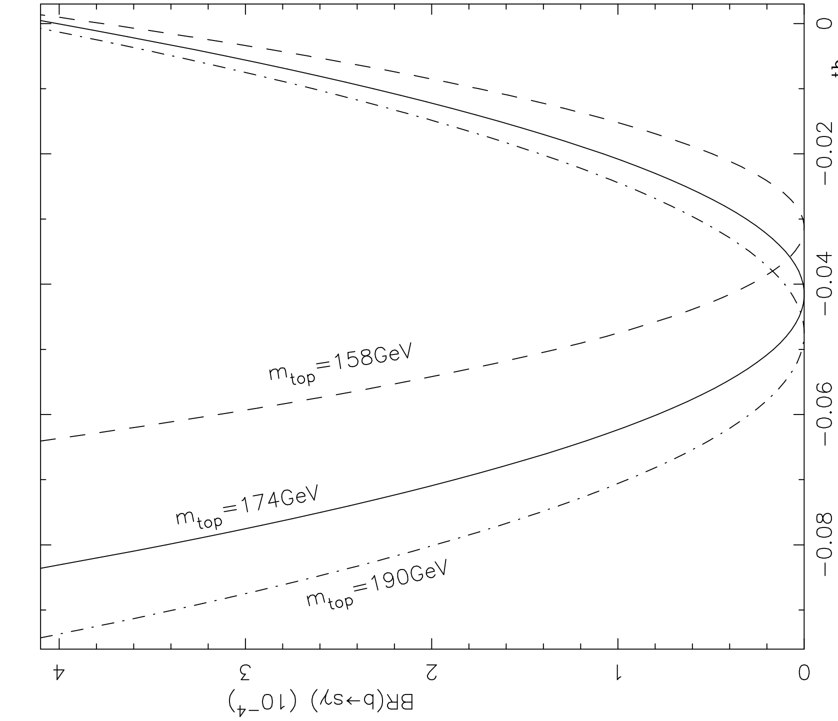
Fig.1 BR($\overline{B} \to X_s\gamma$) as function of f_R^{tb} for different α_s values. $\alpha_s(m_Z) = 0.107,\ 0.117,\ 0.127.$

Fig.2 BR($\overline{B} \to X_s\gamma$) as function of f_R^{tb} for different top quark masses. $m_t = 158,\ 174,\ 190$GeV.



BR(b→s γ) (10⁻⁴)
f_R^{tb}
$\alpha_s(m_Z)$=0.117
$\alpha_s(m_Z)$=0.107
$\alpha_s(m_Z)$=0.127
0
1
2
3
4
−0.08
−0.06
−0.04
−0.02
0

Fig. 1



BR(b→sγ) (10⁻⁴)
f_R^{tb}
m_{top}=158GeV
m_{top}=174GeV
m_{top}=190GeV
0
1
2
3
4
−0.08
−0.06
−0.04
−0.02
0

Fig. 2